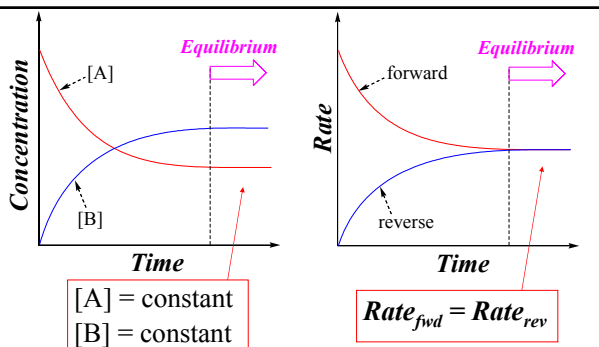


Equilibrium: The Extent of Reactions

- Chemical equilibrium – studies the extent of reactions and the ways it can be altered
- Kinetics and equilibrium are two *different* aspects of chemical reactions (fast reactions may proceed to a great, lesser or a limited extent; same is true for slow reactions)

16.1 The Dynamic Nature of the Equilibrium State

- Chemical equilibrium** – a state in which the concentrations of reactants and products no longer change



- At a given T , the same equilibrium state is reached even if the process is started from different starting points

➤ Equilibrium is **not** a stationary state or a unidirectional process

Example: $A \rightarrow B \rightarrow C$

If the rates of step 1 and step 2 are equal, $[B]$ remains constant $\rightarrow$ not an equilibrium state

➤ **Equilibrium is a dynamic state** achieved by the equalization of the forward and reverse rates of a **reversible** (bidirectional) **process**

Example: $A \leftrightarrow B$

If the rates of the forward and reverse reactions are equal, $[A]$ and $[B]$ remain constant $\rightarrow$ an equilibrium state

$\Rightarrow$ At equilibrium $\rightarrow$ $\text{Rate}_{fwd} = \text{Rate}_{rev}$

Example: $N_2O_4(g; \text{colorless}) \leftrightarrow 2NO_2(g; \text{brown})$

- The reaction can be started from pure $N_2O_4(g; \text{colorless})$ or from pure $NO_2(g; \text{brown})$.
- In both cases at equilibrium, the same light-brown color is reached (the same proportion of N_2O_4 and NO_2 is produced)
- The reaction has a single step mechanism (the forward and reverse reactions are elementary), so at equilibrium:

$$\text{Rate}_f = \text{Rate}_r \rightarrow k_f[N_2O_4] = k_r[NO_2]^2$$

$$\Rightarrow \frac{k_f}{k_r} = K = \frac{[NO_2]^2}{[N_2O_4]}$$

$\rightarrow K$ is a constant which depends on T ($K = 0.211$ at 100°C)

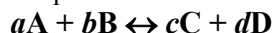
$\rightarrow K$ determines the proportion of N_2O_4 and NO_2 at equilibrium

17.2 The Equilibrium Constant and the Reaction Quotient

The Law of Mass-Action

- Equilibrium constant (K)**

- For a general reaction at equilibrium:



$$K_c = \frac{[C]_e^c [D]_e^d}{[A]_e^a [B]_e^b}$$

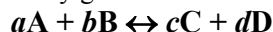
$\rightarrow K_c$ is the **equilibrium constant** in terms of concentration (depends on T and the specific reaction)

$\rightarrow [A]_e$, $[B]_e$, $[C]_e$, and $[D]_e$ are the **equilibrium concentrations** of the reactants and products

$\rightarrow a$, b , c , and d are the **stoichiometric coefficients** of the reactants and products

- Reaction quotient (Q)** – has the same mass-action expression as K

- For a general reaction at any given time:



$$Q_c = \frac{[C]^c [D]^d}{[A]^a [B]^b}$$

$\rightarrow Q_c$ is the **reaction quotient** in terms of concentration (Q_c varies during the reaction)

$\rightarrow [A]$, $[B]$, $[C]$, and $[D]$ are the **current concentrations** of the reactants and products at any given time during the reaction

$\rightarrow$ When the current concentrations become equal to the equilibrium concentrations, $Q_c = K_c$

$\Rightarrow$ At equilibrium $\rightarrow Q = K$

Example: Write the mass action expression for the reaction: $2\text{H}_2(\text{g}) + \text{O}_2(\text{g}) \leftrightarrow 2\text{H}_2\text{O}(\text{g})$

$$Q_c = \frac{[\text{H}_2\text{O}]^2}{[\text{H}_2]^2[\text{O}_2]} \quad \text{At equilibrium} \rightarrow Q_c = K_c$$

- The mass-action expressions for Q and K depend on the form of the chemical equation



or



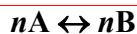
$$Q_c^{\rightarrow} = \frac{[\text{B}]}{[\text{A}]}$$

$$Q_c^{\leftarrow} = \frac{[\text{A}]}{[\text{B}]} = \frac{1}{Q_c^{\rightarrow}}$$

$\Rightarrow Q$ (or K) of the reverse reaction is the reciprocal of Q (or K) of the forward reaction



or



$$Q_c = \frac{[\text{B}]}{[\text{A}]}$$

$$Q_c' = \frac{[\text{B}]^n}{[\text{A}]^n} = (Q_c)^n$$

$\Rightarrow$ Multiplying a reaction by a factor, n , raises Q (or K) to n^{th} power



$$Q_1 = [\text{C}]/[\text{A}][\text{B}]$$



$$Q_2 = [\text{D}]/[\text{C}]$$



$$Q_c = [\text{D}]/[\text{A}][\text{B}]$$

$$Q_1 \times Q_2 = \frac{[\text{C}]}{[\text{A}][\text{B}]} \times \frac{[\text{D}]}{[\text{C}]} = \frac{[\text{D}]}{[\text{A}][\text{B}]} = Q_c$$

$\Rightarrow Q$ (or K) of the sum of two or more reactions is the product of their Q s (or K s)

Example: For the gas phase reaction



K_c is 3.6×10^{-5} at 1200 K. What is K_c' for the reaction

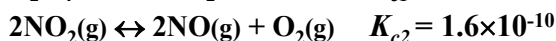


$\rightarrow$ The given reaction has been reversed $\Rightarrow$ take the reciprocal of K_c

$\rightarrow$ The given reaction has been multiplied by 2 $\Rightarrow$ take the square of K_c

$$\Rightarrow K_c' = (1/K_c)^2 = (1/3.6 \times 10^{-5})^2 = 7.7 \times 10^8$$

Example: Given the following two reactions and their K_c s at a certain temperature:



Calculate K_c at this temperature for the reaction



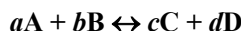
$\rightarrow$ The sum of the given reactions yields the desired reaction $\Rightarrow$ multiply the K_c

$$\Rightarrow K_c = K_{c1} \times K_{c2} = (2.2 \times 10^6) \times (1.6 \times 10^{-10})$$

$$\Rightarrow K_c = 3.5 \times 10^{-4}$$

Connection between Kinetics and Equilibrium

$\triangleright$ For an elementary reaction:



$$\text{Rate}_{fwd} = k_{fwd}[\text{A}]^a[\text{B}]^b \quad \text{and} \quad \text{Rate}_{rev} = k_{rev}[\text{C}]^c[\text{D}]^d$$

$\triangleright$ At equilibrium:

$$\text{Rate}_{fwd} = \text{Rate}_{rev} \rightarrow k_{fwd}[\text{A}]^a[\text{B}]^b = k_{rev}[\text{C}]^c[\text{D}]^d$$

$$\Rightarrow \frac{k_{fwd}}{k_{rev}} = \frac{[\text{C}]^c[\text{D}]^d}{[\text{A}]^a[\text{B}]^b} = Q_c = K_c \Rightarrow K_c = \frac{k_{fwd}}{k_{rev}}$$

$\rightarrow K_c$ is large when the forward reaction is fast and the reverse is slow ($k_{fwd} \gg k_{rev}$)

$\triangleright$ For an overall reaction (sum of elementary reactions), K_c is the product of the K_c s for the individual elementary steps:

$$K_c = K_c' \times K_c'' \times \dots = \frac{k'_{fwd}}{k'_{rev}} \times \frac{k''_{fwd}}{k''_{rev}} \times \dots$$

$\rightarrow$ The magnitude of K_c is an indication of how far a reaction proceeds toward products at a given T

$\rightarrow$ Large K_c ($K_c \gg 1$) – products dominate

$\rightarrow$ Small K_c ($K_c \ll 1$) – reactants dominate

$\rightarrow$ Intermediate K_c ($K_c \sim 1$) – significant amounts of both reactants and products are present at equilibrium

Form of Q and K for Heterogeneous Equilibria

– **Heterogeneous** equilibria – reactants and products in different phases

Example: $2\text{H}_2\text{O}(l) \leftrightarrow 2\text{H}_2(g) + \text{O}_2(g)$

$$Q'_c = \frac{[\text{H}_2(g)]^2 [\text{O}_2(g)]}{[\text{H}_2\text{O}(l)]^2}$$

➤ The concentration of H_2O in pure liquid water is constant at a given temperature

$$[\text{H}_2\text{O}] = \text{mol}/V = (m/MW)/V = d/MW$$

➤ At 25°C :

$$[\text{H}_2\text{O}] = (1.00 \times 10^3 \text{ g/L}) / (18.0 \text{ g/mol}) = 55.6 \text{ mol/L}$$

$$Q'_c \times [\text{H}_2\text{O}(l)]^2 = [\text{H}_2(g)]^2 [\text{O}_2(g)] = Q'_c \times (55.6)^2 = Q_c$$

$$\Rightarrow Q_c = [\text{H}_2]^2 [\text{O}_2]$$

⇒ The molarities of **pure liquids** and **solids** remain constant during the reaction and can be **eliminated** from the expressions for Q and K

Example: $\text{AgCl}(s) \leftrightarrow \text{Ag}^+(aq) + \text{Cl}^-(aq)$

→ $\text{AgCl}(s)$ is a pure solid

$$\Rightarrow Q_c = [\text{Ag}^+][\text{Cl}^-]$$

Example: $\text{S}(s) + \text{O}_2(g) \leftrightarrow \text{SO}_2(g)$

→ $\text{S}(s)$ is a pure solid

$$\Rightarrow Q_c = [\text{SO}_2]/[\text{O}_2]$$

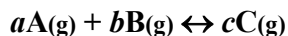
17.3 Equilibria Involving Gases

– Gases are quantified easier through partial pressures rather than molarities

➤ For a gaseous reactant or product, the molarity, C_i , and partial pressure, P_i , are related by the ideal gas law

$$P_i V = n_i RT \quad P_i / RT = n_i / V = C_i$$

➤ For a general reaction



$$Q_c = \frac{[\text{C}]^c}{[\text{A}]^a [\text{B}]^b}$$

$$Q_c = \frac{(P_C / RT)^c}{(P_A / RT)^a (P_B / RT)^b} = \frac{P_C^c}{P_A^a P_B^b} \times \frac{1}{(RT)^{c-(a+b)}}$$

$$Q_c = \frac{P_C^c}{P_A^a P_B^b} \times \frac{1}{(RT)^{\Delta n}} \quad \Delta n = c - (a + b)$$

$$Q_c = Q_p \times \frac{1}{(RT)^{\Delta n}} \quad Q_p = \frac{P_C^c}{P_A^a P_B^b}$$

$$Q_p = Q_c (RT)^{\Delta n} \quad K_p = K_c (RT)^{\Delta n}$$

→ Q_p and K_p are the reaction quotient and equilibrium constant in terms of partial pressures (P is in atm, $R = 0.08206 \text{ L}\cdot\text{atm}/\text{mol}\cdot\text{K}$, T is in K)

→ Δn is the difference between the moles of gaseous products and reactants

$$\Delta n = n(\text{g, products}) - n(\text{g, reactants})$$

Example: For the reaction



$K_c = 2.6 \times 10^{-6}$ at 1000K. What is the partial pressure of CO_2 in the reaction mixture at this temperature?

→ Calculate K_p

$$\rightarrow \Delta n = 1 - 0 = 1$$

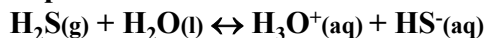
$$\rightarrow K_p = K_c (RT)^{\Delta n} = 2.6 \times 10^{-6} (0.0821 \times 1000) = 2.1 \times 10^{-4}$$

$$\rightarrow K_p = P_{\text{CO}_2} \Rightarrow P_{\text{CO}_2} = 2.1 \times 10^{-4} \text{ atm}$$

➤ Use K_c (or Q_c) if molarities are given, K_p (or Q_p) if partial pressures are given, or K (or Q) for mixed expressions

➤ Omit the subscript “e” from all expressions for K

Example:



$$K = [\text{H}_3\text{O}^+][\text{HS}^-] / P_{\text{H}_2\text{S}}$$

Example: Calculate K_c and K_p for the reaction



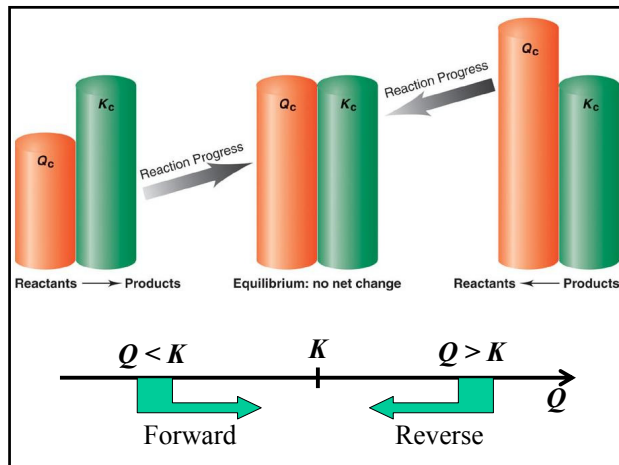
if at 500 K, the equilibrium concentrations of BrCl , Br_2 and Cl_2 are 0.131, 3.51 and 0.156 M, respectively.

$$K_c = \frac{[\text{Br}_2][\text{Cl}_2]}{[\text{BrCl}]^2} = \frac{3.51 \times 0.156}{0.131^2} = 31.9$$

$$\rightarrow \Delta n = 1 + 1 - 2 = 0 \Rightarrow K_p = K_c (RT)^0 = 31.9 \times 1 = 31.9$$

17.4 The Direction of Reaction

- The direction of a reaction can be predicted by comparing the current value of Q to the value of K at the temperature of the reaction
- As the reaction approaches equilibrium, Q approaches K
 - If $Q < K$, the concentration of reactants is too high, while that of products is too low
 $\Rightarrow$ the reaction **proceeds forward**
 - If $Q > K$, the concentration of reactants is too low, while that of products is too high
 $\Rightarrow$ the reaction **proceeds in reverse**
 - If $Q = K$, the reaction is **at equilibrium**



Example: For the reaction



$K_p = 0.98$ at 298 K. Predict the direction of the reaction at 298 K, if the partial pressures of N_2O_4 and NO_2 are 2.4 and 1.2 atm, respectively.

$$Q_p = \frac{P_{\text{NO}_2}^2}{P_{\text{N}_2\text{O}_4}} = \frac{1.2^2}{2.4} = 0.60 \quad K_p = 0.98$$

$\rightarrow Q_p < K_p \Rightarrow$ The reaction proceeds forward to produce more NO_2

17.5 Solving Equilibrium Problems

- Calculation of K_c (or K_p) values from measured equilibrium concentrations (or pressures)
- Calculation of equilibrium concentrations (or pressures) from K_c (or K_p) values
- Equilibrium tables** (“*ice*” tables) – give the initial, *i*, change of, *c*, and equilibrium, *e*, concentrations of reactants and products
 - For a general reaction: $\text{A} + 2\text{B} \leftrightarrow \text{C}$
 - $\rightarrow [\text{A}]_i, [\text{B}]_i, [\text{C}]_i$ – initial concentrations
 - $\rightarrow [\text{A}]_e, [\text{B}]_e, [\text{C}]_e$ – equilibrium concentrations
 - $\rightarrow \Delta[\text{A}], \Delta[\text{B}], \Delta[\text{C}]$, – change in the concentrations
 - $\rightarrow [\text{A}]_e = [\text{A}]_i + \Delta[\text{A}] \rightarrow$ same is valid for B and C



$$\rightarrow \Delta[\text{C}] = +x$$

$$\rightarrow \Delta[\text{A}] = -\Delta[\text{C}] \times (1 \text{ mol A} / 1 \text{ mol C}) = -\Delta[\text{C}] = -x$$

$$\rightarrow \Delta[\text{B}] = -\Delta[\text{C}] \times (2 \text{ mol B} / 1 \text{ mol C}) = -2\Delta[\text{C}] = -2x$$

[]	A	+	2B	$\leftrightarrow$	C
<i>i</i>	$[\text{A}]_i$		$[\text{B}]_i$		$[\text{C}]_i$
<i>c</i>	$-x$		$-2x$		$+x$
<i>e</i>	$[\text{A}]_i - x$		$[\text{B}]_i - 2x$		$[\text{C}]_i + x$

$$\Rightarrow K_c = \frac{[\text{C}]}{[\text{A}][\text{B}]^2} = \frac{([\text{C}]_i + x)}{([\text{A}]_i - x)([\text{B}]_i - 2x)^2}$$

$\rightarrow$ The equation can be used to calculate K_c if x is known or to calculate x if K_c is known

Using Equilibrium Quantities to Calculate K

- If all equilibrium concentrations are given, substitute in the mass action expression to find K
- If the initial concentrations and one equilibrium concentration are given, use an *ice* table to find K

Example: 1.00 mol of NH_3 is sealed in a 1.00 L container and heated to 500 K. Calculate K_c for $2\text{NH}_3(\text{g}) \leftrightarrow \text{N}_2(\text{g}) + 3\text{H}_2(\text{g})$, if at equilibrium the concentration of NH_3 is 0.58 M.

$$\rightarrow [\text{NH}_3]_i = 1.00 \text{ mol} / 1.00 \text{ L} = 1.00 \text{ M}$$

$$\rightarrow [\text{N}_2]_i = [\text{H}_2]_i = 0$$

$$\rightarrow [\text{NH}_3]_e = 0.58 \text{ M}$$

	[I]	$2\text{NH}_3(\text{g}) \leftrightarrow \text{N}_2(\text{g}) + 3\text{H}_2(\text{g})$		
i	1.00	0	0	
c	-2x	+x	+3x	
e	$1.00 - 2x$	x	3x	

$$\rightarrow [\text{NH}_3]_e = 1.00 - 2x = 0.58$$

$$\Rightarrow x = (1.00 - 0.58)/2 = 0.21$$

$$\rightarrow [\text{N}_2]_e = x = 0.21 \text{ M}$$

$$\rightarrow [\text{H}_2]_e = 3x = 0.63 \text{ M}$$

$$\Rightarrow K_c = \frac{[\text{N}_2][\text{H}_2]^3}{[\text{NH}_3]^2} = \frac{[0.21][0.63]^3}{[0.58]^2} = 0.16$$

Using K to Calculate Equilibrium Quantities

- If K and all but one equilibrium concentrations are given, substitute in the mass action expression for K to find the unknown concentration
- If the initial concentrations and K are given, use an *ice* table to find the equilibrium concentrations

Example: 0.50 mol of HI is sealed in a 2.0 L reactor and heated to 700°C. Calculate the equilibrium concentrations of all species if at 700°C, $K_c = 0.022$ for $2\text{HI(g)} \leftrightarrow \text{H}_2\text{(g)} + \text{I}_2\text{(g)}$.

$$\rightarrow [\text{HI}]_i = 0.50 \text{ mol}/2.0 \text{ L} = 0.25 \text{ M}$$

$$\rightarrow [\text{I}_2]_i = [\text{H}_2]_i = 0$$

	[I]	2HI(g) ↔ H _{2(g)} + I _{2(g)}	$K_c = \frac{[\text{H}_2][\text{I}_2]}{[\text{HI}]^2} = 0.022$
i	0.25	0	
c	-2x	+x	
e	0.25 - 2x	x	

$$\frac{x \cdot x}{(0.25 - 2x)^2} = 0.022 \Rightarrow \sqrt{\frac{x^2}{(0.25 - 2x)^2}} = \sqrt{0.022}$$

$$\frac{x}{0.25 - 2x} = \sqrt{0.022} \Rightarrow x = \sqrt{0.022} \times 0.25 - \sqrt{0.022} \times 2x$$

$$x + 0.297x = 0.0371 \Rightarrow x = \frac{0.0371}{1 + 0.297} = 0.029$$

$$\Rightarrow [\text{H}_2]_e = [\text{I}_2]_e = x = 0.029 \text{ M}$$

$$\Rightarrow [\text{HI}]_e = 0.25 - 2x = 0.25 - 2 \times 0.029 = 0.19 \text{ M}$$

➤ Using the quadratic formula

Example: 0.50 mol HI and 0.30 mol H₂ are sealed in a 2.0 L reactor and heated to 700°C. Calculate the equilibrium concentrations of all species if at 700°C, $K_c = 0.022$ for $2\text{HI(g)} \leftrightarrow \text{H}_2\text{(g)} + \text{I}_2\text{(g)}$.

$$\rightarrow [\text{HI}]_i = 0.50 \text{ mol}/2.0 \text{ L} = 0.25 \text{ M}$$

$$\rightarrow [\text{H}_2]_i = 0.30 \text{ mol}/2.0 \text{ L} = 0.15 \text{ M}$$

$$\rightarrow [\text{I}_2]_i = 0$$

	[I]	2HI(g) ↔ H _{2(g)} + I _{2(g)}	$K_c = \frac{[\text{H}_2][\text{I}_2]}{[\text{HI}]^2} = 0.022$
i	0.25	0.15	
c	-2x	+x	
e	0.25 - 2x	0.15 + x	

$\frac{(0.15 + x)x}{(0.25 - 2x)^2} = 0.022 \Rightarrow \frac{0.15x + x^2}{0.25^2 - 2 \times 0.25 \times 2x + 4x^2} = 0.022$
$0.15x + x^2 = 0.022 \times 0.25^2 - 0.022 \times 4 \times 0.25x + 0.022 \times 4x^2$
$0.15x + x^2 = 0.00138 - 0.022x + 0.088x^2$
$0.912x^2 + 0.172x - 0.00138 = 0$
$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \text{The } (-) \text{ solution is meaningless}$
$x = \frac{-0.172 + \sqrt{0.172^2 - 4 \times 0.912 \times (-0.00138)}}{2 \times 0.912} = 0.00768$
$\Rightarrow [\text{I}_2]_e = x = 0.0077 \text{ M}$
$\Rightarrow [\text{H}_2]_e = 0.15 + x = 0.15 + 0.00768 = 0.16 \text{ M}$
$\Rightarrow [\text{HI}]_e = 0.25 - 2x = 0.25 - 2 \times 0.00768 = 0.23 \text{ M}$

➤ Using simplifying assumptions

Example: A mixture of 0.060 M N₂ and 0.040 M H₂ is heated to a temperature where $K_c = 0.0010$ for $\text{N}_2\text{(g)} + 3\text{H}_2\text{(g)} \leftrightarrow 2\text{NH}_3\text{(g)}$. Calculate the equilibrium concentration of NH₃.

$$\rightarrow [\text{N}_2]_i = 0.060 \text{ M}$$

$$\rightarrow [\text{H}_2]_i = 0.040 \text{ M}$$

$$\rightarrow [\text{NH}_3]_i = 0$$

	[I]	N _{2(g)} + 3H _{2(g)} ↔ 2NH _{3(g)}	$K_c = \frac{[\text{NH}_3]^2}{[\text{N}_2][\text{H}_2]^3} = 0.0010$
i	0.060	0.040	
c	-x	-3x	
e	0.060 - x	0.040 - 3x	

$$\frac{(2x)^2}{(0.060 - x)(0.040 - 3x)^3} = 0.0010$$

→ x is expected to be very small (K_c is quite small, so the product molarity, $2x$, should be small too)
 ⇒ **Assumption** → x can be neglected in summations
 ⇒ $0.060 - x \approx 0.060$ and $0.040 - 3x \approx 0.040$

$$\frac{4x^2}{(0.060)(0.040)^3} = 0.0010 \Rightarrow x = \sqrt{\frac{0.0010 \times 0.060 \times 0.040^3}{4}}$$

$$x = 3.1 \times 10^{-5}$$

→ Must check the assumption ($x \ll 0.06$ and $3x \ll 0.04$)
 → The **assumption is justified** since x and $3x$ are less than 5% of 0.060 and 0.040 → The **5% rule**
 ⇒ $[\text{NH}_3]_e = 2x = 2 \times (3.1 \times 10^{-5}) = 6.2 \times 10^{-5} \text{ M}$

➤ Simplifying assumptions are not always justified

Example: A mixture of **0.060 M** N_2 and **0.040 M** H_2 is heated to a temperature where $K_c = 10$. for $\text{N}_2(\text{g}) + 3\text{H}_2(\text{g}) \leftrightarrow 2\text{NH}_3(\text{g})$. Calculate the equilibrium concentration of NH_3 .
 → Using the assumption from the previous problem leads to:

$$\frac{4x^2}{(0.060)(0.040)^3} = 10. \Rightarrow x = \sqrt{\frac{10 \times 0.060 \times 0.040^3}{4}}$$

$$x = 0.0031$$

→ The **assumption is not justified** since x and $3x$ are more than 5% of 0.060 and 0.040
 $(3 \times 0.0031 / 0.040) \times 100\% = 23\% \text{ error}$

⇒ We must solve the equation without assumptions

➤ **Successive approximation**
 → Using the same formula as in the previous problem without neglecting x and $3x$ leads to:

$$x_{n+1} = \sqrt{\frac{10 \times (0.060 - x_n)(0.040 - 3x_n)^3}{4}}$$

→ This formula allows the calculation of the **(n+1)st** approximation for x from the **nth** approximation
 → For **n=0**, assume $x_0 = 0$ (x is expected to be small)
 → For **n=1** (1st iteration)

$$x_1 = \sqrt{\frac{10 \times (0.060 - 0)(0.040 - 3 \times 0)^3}{4}} = 0.0031$$

→ For **n=2** (2nd iteration)

$$x_2 = \sqrt{\frac{10 \times (0.060 - 0.0031)(0.040 - 3 \times 0.0031)^3}{4}} = 0.0020$$

→ For **n=3** (3rd iteration)

$$x_3 = \sqrt{\frac{10 \times (0.060 - 0.0020)(0.040 - 3 \times 0.0020)^3}{4}} = 0.0024$$

→ For **n=4** (4th iteration)

$$x_4 = \sqrt{\frac{10 \times (0.060 - 0.0024)(0.040 - 3 \times 0.0024)^3}{4}} = 0.0023$$

→ For **n=5** (5th iteration) → $x_5 = 0.0023$
 → Since $x_5 \approx x_4$ (convergence) ⇒ $x = 0.0023$
 ⇒ $[\text{NH}_3]_e = 2x = 2 \times (0.0023) = 0.0046 \text{ M}$

Equilibrium Calculations for Reactions with Unknown Direction

Example: **0.50 mol** H_2 , **0.50 mol** I_2 and **0.50 mol** HI are mixed in a **1.0 L** container and heated to a temperature where $K_c = 0.45$ for the reaction $\text{H}_2(\text{g}) + \text{I}_2(\text{g}) \leftrightarrow 2\text{HI}(\text{g})$. Calculate $[\text{HI}]$ at equilibrium.

→ $[\text{H}_2]_i = [\text{I}_2]_i = [\text{HI}]_i = 0.50 \text{ mol} / 1.0 \text{ L} = 0.50 \text{ M}$
 → Since all reactants and products are present initially, the direction of the reaction must be determined first
 ⇒ Q_c must be calculated and compared to K_c

$$Q_c = \frac{[\text{HI}]^2}{[\text{H}_2][\text{I}_2]} = \frac{0.50^2}{0.50 \times 0.50} = 1 > K_c$$

→ $Q_c > K_c \Rightarrow$ the reaction proceeds to the left

	$\text{H}_2(\text{g})$	+	$\text{I}_2(\text{g})$	$\leftrightarrow$	$2\text{HI}(\text{g})$	
i	0.50		0.50		0.50	$K_c = \frac{[\text{HI}]^2}{[\text{H}_2][\text{I}_2]} = 0.45$
c	+x		+x		-2x	
e	0.50 + x		0.50 + x		0.50 - 2x	
$i + c = e$						

$$\frac{(0.50 - 2x)^2}{(0.50 + x)(0.50 + x)} = 0.45 \Rightarrow \sqrt{\frac{(0.50 - 2x)^2}{(0.50 + x)^2}} = \sqrt{0.45}$$

$$\frac{(0.50 - 2x)}{(0.50 + x)} = \sqrt{0.45} \Rightarrow (0.50 - 2x) = \sqrt{0.45} \times (0.50 + x)$$

$$0.50 - 0.67 \times 0.50 = 2x + 0.67x \Rightarrow x = \frac{0.165}{2 + 0.67} = 0.062$$

⇒ $[\text{HI}]_e = 0.50 - 2x = 0.50 - 2 \times 0.062 = 0.38 \text{ M}$

17.6 Reaction Conditions and the Equilibrium State

- **Le Chatelier's principle** – when a system at equilibrium is disturbed, the equilibrium “shifts” in a direction that minimizes the effect of the disturbance

– A chemical system can be disturbed by changing the values of Q or K so that temporarily $Q \neq K$

- Changing concentrations of reactants or products (Q changes)
- Changing pressure for gas reactions (Q changes)
- Changing temperature (K changes)

Changing Concentration

- If the concentration increases, the system reacts to consume some of it; If the concentration decreases, the system reacts to produce some of it

➤ Adding reactants or removing products

→ The equilibrium **shifts toward the products** in order to consume the added reactants or generate the removed products

→ Q decreases → $Q < K$ → reaction **shifts forward**

➤ Adding products or removing reactants

→ The equilibrium **shifts toward the reactants** in order to consume the added products or generate the removed reactants

→ Q increases → $Q > K$ → reaction **shifts in reverse**

Example: Given $N_2(g) + 3H_2(g) \leftrightarrow 2NH_3(g)$. How can the yield of NH_3 be increased by manipulating the concentrations of reactants and products?

- Add more N_2 and H_2
- Remove some NH_3

Changing Pressure or Volume

- Affects the concentrations of gaseous reactants and products (changes Q)

➤ Changing the partial pressure of a gaseous reactant or product

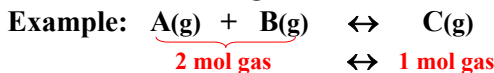
→ The concentration of the reactant or product changes and the equilibrium shifts accordingly as described before

➤ Changing the total pressure of the reaction mixture by changing its volume

➤ Compression ($\uparrow P$ by $\downarrow V$)

→ The equilibrium shifts in a direction that consumes gases and relieves the pressure

⇒ The equilibrium **shifts toward the side with fewer moles of gas**



$$Q_p = \frac{P_c}{P_A P_B} = \frac{(n_c RT / V)}{(n_A RT / V)(n_B RT / V)} = \frac{n_c}{n_A n_B} \times \frac{V}{RT}$$

⇒ As V is reduced, Q decreases ($Q < K$) and the reaction shifts forward

➤ Expansion ($\downarrow P$ by $\uparrow V$)

→ The equilibrium shifts in a direction that produces more gases and increases the pressure

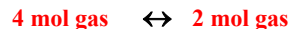
⇒ The equilibrium **shifts toward the side with more moles of gas**

➤ Compression and expansion do not affect reactions in which the number of moles of gases is the same on both sides of the equation

➤ Changing the total pressure of the reaction mixture by adding an inert gas

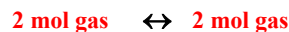
→ The equilibrium **is not affected** because the partial pressures and concentrations of the components do not change

Example: Given $N_2(g) + 3H_2(g) \leftrightarrow 2NH_3(g)$. How can the yield of NH_3 be increased by changing the pressure (volume) of the reaction mixture?



⇒ Increasing the pressure (compression) shifts the equilibrium to the right toward less moles of gas (4 mol → 2 mol) and improves the yield of NH_3

Example: Given $Cl_2(g) + H_2(g) \leftrightarrow 2HCl(g)$. What is the effect of increasing the volume of the reaction container?



⇒ Increasing the volume (expansion) has no effect on the reaction since the number of moles of gas is the same on both sides of the equation

Changing Temperature

- Affects the value of K

➤ **Increasing T** by adding heat to the reaction mixture **favors the endothermic reaction** which consumes the added heat

➤ **Decreasing T** by removing heat from the reaction mixture **favors the exothermic reaction** which produces heat

Example: $A + B \leftrightarrow C + D + \text{heat}$ $\Delta H < 0$

→ The forward reaction is exothermic, while the reverse reaction is endothermic

⇒ Increasing T favors the endothermic reaction so the reaction shifts in reverse

➤ Changing T changes the value of K

➤ Increasing T increases K for endothermic reactions

➤ Increasing T decreases K for exothermic reactions

→ Increasing T increases more the rate constant of the endothermic reaction (which has higher activation energy) → $K = k_{fwd}/k_{rev}$ ⇒ If the forward reaction is endothermic, K increases

Example: Given $N_2(g) + 3H_2(g) \leftrightarrow 2NH_3(g)$ with $\Delta H = -92$ kJ/mol. How can the yield of NH_3 be increased by manipulating the temperature?

→ The forward reaction is exothermic

⇒ Lowering the temperature facilitates the forward reaction and improves the yield of NH_3

➤ The T dependence of K is given by the van't Hoff equation

$$\ln \frac{K_2}{K_1} = -\frac{\Delta H_{rxn}^o}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$$

→ The equation allows the calculation of K at one temperature knowing the value of K at another temperature and ΔH^o of the reaction

Example: Given $N_2(g) + 3H_2(g) \leftrightarrow 2NH_3(g)$ with $\Delta H = -92$ kJ/mol. If $K = 1$ at C , what is K at C ?